Locally AH-algberas

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3. (Dadarlat-Eilers) Inductive limits of AH-algebras may not be AH-algebras.
4. Combing a result of L and Villadsen, Winter gave the following:

A unital simple separable locally AH-algebras with real rank zero, stable rank one and weakly unperforated K_0-group is in fact an AH-algebra with no dimension growth.

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Question 2: What about tracial rank 2, 3, ...?

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(3) The homomorphism ϕ_2 factors through $C([0, 1])$ as

$$\phi_2 : C(X) \to C[0; 1] \to Q_2 M_k(C(Y))Q_2 :$$

Furthermore, if $Y \neq pt$, then the map from $C([0, 1])$ to $Q_2 M_k(C(Y))Q_2$ is injective;

(4) The set $(\phi_0 + \phi_1)(F)$ is approximately constant to within ϵ;
(5) $Q_1 = p_1 + \cdots + p_n$ with $J[Q_0] \le [p_i]$ $(i = 1, ..., n)$, ϕ_0 is dened by $\phi_1(f) = Q_0\phi(f)Q_0$, and ϕ_1 is dened by

$$\phi_1(f) = \sum_{j=1}^{n} f(x_i)p_i$$

for all $f \in C(X)$; where $p_0, p_1, ..., p_n$ are mutually orthogonal projections and $\{x_1, x_2, ..., x_n\} \subset X$ is an ϵ-dense subset of X.

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If we could, we should be able to answer Question 1 and 2.
It has been an open question ever since Gong's decomposition first appeared around 1997.

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Then

$$f_\epsilon(a) \lesssim f_{\epsilon^2/2^9}(pap + (1-p)a(1-p)). \tag{e 0.2}$$

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Then $A \cong B$.
Moreover, A is isomorphic to a unital simple AH-algebra with no dimension growth.

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$$[\phi]|_{\mathcal{P}} = [\psi]|_{\mathcal{P}} = [h]|_{\mathcal{P}} \qquad \text{(e 0.4)}$$

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$$\mu_{\tau\circ\phi}(O_r) \geq \sigma_i, \;\; \mu_{\tau\circ\psi}(O_r) \geq \sigma_i, \tag{e 0.5}$$

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there exists a unitary $W \in M_n(C([0,1]))$ such that

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\{[L_n]\} &= \kappa, \\
\lim_{n\to\infty} \sup_{\tau\in T(A)} |\tau \circ L_n(c) - \gamma(\tau)(c)| &= 0 \ \textit{for all} \ c \in C_{s.a.} \ \text{and} \\
\lim_{n\to\infty} \|L_n(ab) - L_n(a)L_n(b)\| &= 0 \ \textit{for all} \ a, b \in C \ \text{and} \\
\lim_{n\to\infty} \mathrm{dist}(\eta([u]), \overline{\langle L_n(u)\rangle}^{\dagger}) &= 0 \ \textit{for all} \ u \in U_c(K_1(C)).
\end{aligned}$$

Thank you for listening!